**Logo

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**MATH201 - Calculus-I**

**Homework Assignment #4**

**Due day: 8/6/2023**

**Instruction:**

1. **Push the answer sheet to GitHub in word file**
2. **Overdue homework submission could not be accepted.**
3. **Takes academic honesty and integrity seriously (Zero Tolerance of Cheating & Plagiarism)**
4. **(a) Graph in Excel the function in the viewing rectangle [-2π, 2π] by [-4, 4]. What slope does the graph appear to have at the origin?**

A graph of a function

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When we look at the graph, the graph appears to be linear around the origin. The slope is likely to be close to the derivative of the function at x = 0.

The derivative of the given function is:

Evaluating at x + 0, we get:

So, the graph appears to have a slope of 0 at the origin, regardless of the high-frequency oscillations.

**(b) Zoom in to the viewing window [-0.4, 0.4] by [-0.25, 0.25] in Excel and estimate the value of . Does this agree with your answer from part (a)?**

A graph with numbers and lines

Description automatically generated

Previously, we estimated the value of The slope is likely close to the derivative of the function at x = 0.

The derivative of the given function is:

Evaluating at x = 0:

So, by zooming in on the origin, we can see that the slope appears to be 0. This agrees with the analytical answer from part (a), where the function's derivative at x = 0 was found to be 0.

**(c) Now zoom in to the viewing window [-0.008, 0.008] by [-0.005, 0.005] in Excel. Do you wish to revise your estimate for ?**

A screenshot of a graph

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With such a close zoom, we’re able to see the details of the graph near the origin. The slope at the origin still appears to be 0, which is consistent with the analytical answer from part (a).

In theory, the derivative at x = 0 is 0, so the graph should agree with this.

1. **Graph in Excel the function . Zoom in repeatedly, first toward the point (-1, 0) and then toward the origin. What is different about the behavior of *f* in the vicinity of these two points? What do you conclude about the differentiability of *f* ?**

Zooming in toward the point (-1, 0):

A graph with a line going up

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The function has a cusp at this point. The slope changes abruptly, and the derivative doesn’t exist.

Zooming in toward the Origin:

A graph with a dotted line

Description automatically generated

The function is smooth and differentiable at the origin.

The behaviour near these two points highlights that the function is not differentiable at x = -1 but is differentiable at the origin. This difference in behaviour illustrates how a function can be smooth at some points and have discontinuities in its derivative at others.

The cusp at x = -1 results from the square root function combined with the absolute value inside it. This creates a sharp bend in the graph, making the derivative undefined at that point.

1. **The left-hand and right-hand derivatives of *f* at *a* are defined by**

**and**

**if these limits exist. Then exists if and only if these one-sided derivatives exist and are equal.**

1. **Find and for the function**

**A graph on a white sheet

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1. **Sketch the graph of *f*.**

**A graph on a white sheet

Description automatically generated**

Excel doesn’t perfectly represent the vertical asymptote and the discontinuity in the derivative at x = 4, since it connects the points with straight lines. However, the above provides a practical way to visualize the function.

**(c) Where is *f* discontinuous?**

At

* From the left:
* From the right:
* Since the values are different, there is a discontinuity at

At :

* From the left: f(
* From the right: f(
* Since the values are the same, there is no discontinuity at .

Hence, the function is discontinuous at .

**(d) Where is *f* not differentiable?**

At

* Since there is a discontinuity at the function is not differentiable at this point.

At

* From the left: The derivative is the slope of the line
* From the right: The derivative is more complex due to the hyperbola and it differs from the left-hand derivative.
* Since the left-hand and right-hand derivatives are not equal, the function is not differentiable at

So, the function is not differentiable at

1. **If *f* is a differentiable function and , use the definition of a derivative to show that**

The definition of derivative states that the derivative of a function at a point is given by:

Let’s apply this definition to

We’ll now use the product rule for limits, which allows us to split the limit of a product into the product of the limits:

So we have shown that .

1. **Boyle’s Law states that when a sample of gas is compressed at a constant temperature, the pressure *P* of the gas is inversely proportional to the volume *V* of the gas.**
   1. **Suppose that the pressure of a sample of air that occupies *0.106* at *25℃* is *50 kPa*. Write *V* as a function of *P*.**

Where is a constant.

Given that the pressure of a sample of air is when it occupies a volume of we can find the value of the constant :

.

Now, we can write as a function of using the relationship:

Substituting the value of , we can express the volume as a function of pressure:

This equation represents the relationship between the volume and pressure for the given sample of air, according to Boyle’s Law.

* 1. **Calculate when *P = 50 kPa*. What is the meaning of the derivative? What are its units?**

We have the expression for the volume as a function of pressure :

Now, we want to find the derivative of with respect to and evaluate it at = 50 kPa.

The derivative represents the rate of change of the volume with respect to the pressure, and it can be calculated as:

Let’s calculate this derivative and evaluate it at = 50 kPa.

Given the expression for the volume:

The derivative with respect to

Now, we can evaluate this derivative at :

=

**Meaning of the Derivative:**

The derivative represents the rate of change of the volumne with respect to the pressure. Since it’s negative, it means that as the pressure increases, the volume decreases, which aligns with Boyle’s Law.

**Units of the Derivative:**

The units of the derivative are cubic meters per kilopascal (, which are the units of volume divided by the units of pressure.

1. **Car tires need to be inflated properly because overinflation or underinflation can cause premature tread wear. The data in the table show tire life *L* (in thousands of miles) for a certain type of tire at various pressures *P* (in ).**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***P*** | ***26*** | ***28*** | ***31*** | ***35*** | ***38*** | ***42*** | ***45*** |
| ***L*** | ***50*** | ***66*** | ***78*** | ***81*** | ***74*** | ***70*** | ***59*** |

* 1. **Use a calculator to model tire life with a quadratic function of the pressure.**

To model the tire life as a quadratic function of the pressure , we can use the general quadratic function of the form:

Given the data points for

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***P*** | ***26*** | ***28*** | ***31*** | ***35*** | ***38*** | ***42*** | ***45*** |
| ***L*** | ***50*** | ***66*** | ***78*** | ***81*** | ***74*** | ***70*** | ***59*** |

We can use three of these points to set up a system of equations to solve for the coefficients However, since there are more than three data points, a perfect fit might not be possible. Instead, we will typically use a method like least squares to find the best fitting quadratic curve.

For simplicity, let’s use the first three data points to set up our equations:

Using

Using

Using

The quadratic model for tire life in terms of pressure using the first three data points is:

This model approximaes the relationship between tire life and pressure for the given data. However, we must note that this is an approximation based on only three data points, and it might not perfectly fit all the data points in the table.

* 1. **Use the model to estimate when *P = 30* and when *p = 40*. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?**

The derivative of the tire life with respect to pressure denoted represents the rate of change of the tire life with respect to the pressure. It tells us how much the tire life changes for a small change in pressure.

Let’s first find the expression for using the quadratic model:

Then, we will evaluate this derivative at

**Meaning of the Derivative:**

**Positive Value:** If the derivative is positive, it means that as the pressure increases, the tire life also increases.

**Negative Value:** If the derivative is negative, it means that as the pressure increases, the tire life decreases.

**Units of the Derivative:**

The units of the derivative will be the units of tire life (thousands of miles) divided by the units of pressure (, so the units are thousnads of miles per .

The derivatives of tire life with respect to pressure at the given points are:

* At
* At

**Meaning and Significance of the Derivatives:**

* At , the positive derivative indicates that an increase in pressure would lead to an increase in tire life.
* , the negative derivative indicates that an increase in pressure would lead to a decrease in tire life.

These derivatives highlight the sensitivity of tire life to changes in pressure and can guide decisions about optimal tire inflation. If the pressure is too low (around , increasing it can prolong the tire life, while if the pressure is too high (around , further increasing it can reduce the tire life. The optimal pressure would likely be somewhere in between, where the derivative is close to zero.